

Date for handing in: Oct. 22

Exercises – 1:

1. Use Gaussian elimination with back substitution to solve the following system:

$$\begin{cases} 4x_2 - 3x_3 = 3 \\ -x_1 + 8x_2 - 6x_3 = 5 \\ -x_1 + 7x_2 - 5x_3 = 4 \end{cases}$$

2. Solve the following system:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 + x_2 + 3x_3 + 3x_4 = 3 \\ x_1 + x_2 + 2x_3 + 3x_4 = 3 \\ x_1 + 3x_2 + 3x_3 + 3x_4 = 4 \end{cases}$$

3. Prove that the set $\mathbb{R}^2$ is a vector space if the operations '+' and '·' have their usual meaning.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix} \quad r \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} rx_1 \\ rx_2 \end{pmatrix}$$

You should check all of the conditions in the definition.

4. This subset of $\mathbb{R}^3$ that is a plane through the origin

$$P = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid x + y + z = 0 \right\}$$

is a vector space if '+' and '·' are interpreted in this way:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{pmatrix} \quad r \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} rx \\ ry \\ rz \end{pmatrix}$$

5. The set of polynomials with real coefficients

$$\{a_0 + a_1x + \dots + a_nx^n \mid n \in \mathbb{N} \text{ and } a_0, \dots, a_n \in \mathbb{R}\}$$

makes a vector space when given the natural '+'

$$\begin{aligned} (a_0 + a_1x + \dots + a_nx^n) + (b_0 + b_1x + \dots + b_nx^n) \\ = (a_0 + b_0) + (a_1 + b_1)x + \dots + (a_n + b_n)x^n \end{aligned}$$

and '·'.

$$r \cdot (a_0 + a_1x + \dots + a_nx^n) = (ra_0) + (ra_1)x + \dots + (ra_n)x^n$$

6. Give the zero vector from each of these vector spaces.
- (a) The space of degree three polynomials under the natural operations
 - (b) The space of 2×4 matrices
 - (c) The space $\{f : [0..1] \rightarrow \mathbb{R} \mid f \text{ is continuous}\}$
 - (d) The space of real-valued functions of one natural number variable X
7. Find the additive inverse, in the vector space, of the vector.

(a) In $\mathcal{P}_3$, the vector $-3 - 2x + x^2$

(b) In the space of 2×2 matrices with real number entries under the usual matrix addition and scalar multiplication,

$$\begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix}$$

(c) In $\{ae^x + be^{-x} \mid a, b \in \mathbb{R}\}$, a space of functions of the real variable x under the natural operations, the vector $3e^x - 2e^{-x}$.

8. Which of these subsets of the vector space of 2×2 matrices are subspaces under the inherited operations? For each one that is a subspace, parametrize its description. For each that is not, give a condition that fails.

(a) $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a, b \in \mathbb{R} \right\}$

(b) $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a + b = 0 \right\}$

(c) $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a + b = 5 \right\}$

(d) $\left\{ \begin{pmatrix} a & c \\ 0 & b \end{pmatrix} \mid a + b = 0, c \in \mathbb{R} \right\}$

9.

Is this a subspace of $\mathcal{P}_2$: $\{a_0 + a_1x + a_2x^2 \mid a_0 + 2a_1 + a_2 = 4\}$? If so, parametrize its description.

10.

Decide if the vector lies in the span of the set, inside of the space.

(a) $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}, \text{ in } \mathbb{R}^3$

(b) $x - x^3, \{x^2, 2x + x^2, x + x^3\}, \text{ in } \mathcal{P}_3$

(c) $\begin{pmatrix} 0 & 1 \\ 4 & 2 \end{pmatrix}, \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 2 & 3 \end{pmatrix} \right\}, \text{ in } \mathcal{M}_{2 \times 2}$